DISTRIBUSI BERSAMA

BAB IV

DISTRIBUSI BERSAMA Peubah Acak Diskrit

Jika X_1, X_2 peubah acak diskrit, maka fungsi peluang bersama peubah acak X_1, X_2 adalah

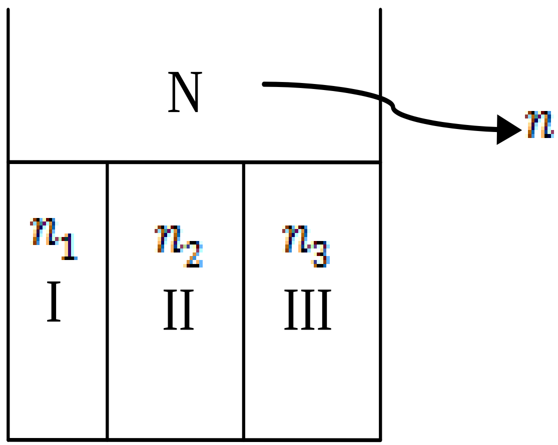
$$f(x_1, x_2) = P(X_1 = x_1, X_2 = x_2)$$

Jika $X_1, X_2, \ldots, X_n$ peubah acak diskrit, maka fungsi peluang bersama $X_1, X_2, \ldots, X_n$ adalah

$$f(x_1, x_2, \ldots, x_n) = P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n)$$

Contoh distribusi bersama

1. **Distribusi Hipergeometri yang diperluas**



X_1: banyak objek I yang terambil

X_2: banyak objek II yang terambil

$$f(x_1, x_2) = P(X_1 = x_1, X_2 = x_2)$$

$$= \frac{\binom{n_1}{x_1}\binom{n_2}{x_2}\binom{n_3}{n - x_1 - x_2}}{\binom{N}{n}}$$

Distribusi Multinomial

$X \sim MULT(n, p_1, p_2, \dots, p_k)$

$$f(x_1, x_2, x_3, \dots, x_k) = \frac{n!}{x_1!\,x_2!\,x_3!\dots x_{k+1}!}\, p_1^{x_1}\ p_2^{x_2}\ p_3^{x_3}\ \dots p_{k+1}^{x_{k+1}}$$

Sifat- sifat

Jika X_1, X_2 peubah acak diskrit dengan fungsi peluang bersama $f(x_1, x_2)$, maka

$$\sum_{x_2}\sum_{x_1} f(x_1, x_2) = 1$$

Jika $X_1, X_2, ..., X_k$ peubah acak diskrit dengan fungsi peluang bersama $f(x_1, x_2, ..., x_k)$ maka, $\sum\sum ... \sum f(x_1, x_2, ..., x_k) = 1$

Fungsi Peluang Batas/ Marginal

Jika X_1, X_2 peubah acak diskrit dengan fungsi peluang bersama $f(x_1, x_2)$ maka fungsi peluang marginal dari X_1 dan X_2 berturut- turut adalah :

$$f_{X_1}(x_1) = \sum_{x_2} f(x_1, x_2)$$

$$f_{X_2}(x_2) = \sum_{x_1} f(x_1, x_2)$$

Fungsi Distribusi Kumulatif Bersama

Misal X_1, X_2 peubah acak diskrit dengan fungsi peluang bersama $f(x_1, x_2)$. Fungsi distribusi kumulatif (CDF) bersama X_1, X_2 didefinisikan sebagai

$$F(x_1, x_2) = P(X_1 \le x_1, X_2 \le x_2) = \sum_{t_2 \le x_2} \sum_{t_1 \le x_1} f(t_1, t_2)$$

Independen

X_1 dan X_2 dikatakan saling bebas/ independen jika

$$f(x_1, x_2) = f(x_1).f(x_2)$$

$$F(x_1, x_2) = F(x_1).F(x_2)$$

Distribusi Bersyarat

$$\text{Ingat}: P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$f(x \mid y) = \frac{f(x, y)}{f_Y(y)} \quad \text{dengan} \quad f(x, y) = f_Y(y).f(x \mid y) = f_X(x).f(y \mid x)$$

$$\text{Jika } X \text{ dan } Y \text{ independen maka } f(x \mid y) = f_X(x)$$

Latihan

2. In Exercise 2 of Chapter 3, a game consisted of rolling a die and tossing a coin. If X denotes the number of spots showing on the die plus the number of heads showing on the coin, and if Y denotes just the number of spots showing on the die, tabulate the joint pdf of X and Y.

		Dadu					
		1	2	3	4	5	6
Koin	0	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)
	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)

Misalkan

X : jumlah mata dadu dan gambar yang muncul

Y : mata dadu yang muncul, maka nilai-nilai:

$x = 1,2,3,4,5,6,7$

$y = 1,2,3,4,5,6$

Fungsi peluang bersama X dan Y $f(x,y)$

y \ x	1	2	3	4	5	6	7	
1	$f(1,1)$	$f(2,1)$	$f(3,1)$					
2								
3								
4								
5								
6								
							$\sum$	

y \ x	1	2	3	4	5	6	7	
1	1/12	1/12	0	0	0	0	0	2/12
2	0	1/12	1/12	0	0	0	0	2/12
3	0	0	1/12	1/12	0	0	0	2/12
4	0	0	0	1/12	1/12	0	0	2/12
5	0	0	0	0	1/12	1/12	0	2/12
6	0	0	0	0	0	1/12	1/12	2/12
							$\sum$	1

7. Suppose that X_1 and X_2 are discrete random variables with joint pdf of the form

$$f(x_1, x_2) = c(x_1 + x_2) \qquad x_1 = 0, 1, 2;\ x_2 = 0, 1, 2$$

and zero otherwise. Find the constant c.

Sifat: $\sum_{x_2}\sum_{x_1} f(x_1, x_2) = 1$

$$\sum_{x_2}\sum_{x_1} f(x_1, x_2) = 1$$

$$\Leftrightarrow f(0,0) + f(1,0) + f(2,0) + f(0,1) + f(1,1) + f(2,1) + f(0,2) + f(1,2) + f(2,2) = 1$$

$$\Leftrightarrow c(0+0) + c(1+0) + c(2+0) + c(0+1) + c(1+1) + c(2+1) + c(0+2) + c(1+2) + c(2+2) = 1$$

$$\Leftrightarrow 0 + c + 2c + c + 2c + 3c + 2c + 3c + 4c = 1$$

$$\Leftrightarrow 18c = 1$$

$$\Leftrightarrow c = \frac{1}{18}$$

8. If X and Y are discrate random variables with joint pdf

$$f(x,y)=\begin{cases} c\dfrac{2^{x+y}}{x!\,y!} & ;x=0,1,2,\ldots,\ y=0,1,2,\ldots \\ 0 & ;x,y \text{ otherwise} \end{cases}$$

a. Find the constant c

$$\sum_{y}\sum_{x} f(x,y)=1$$

$$\Leftrightarrow c\sum\sum\frac{2^x 2^y}{x!\,y!}=1$$

$$\Leftrightarrow c\sum_{x=0}^{\infty}\frac{2^x}{x!}\sum_{y=0}^{\infty}\frac{2^y}{y!}=1$$

$$\Leftrightarrow c.e^2.e^2=1$$

$$\Leftrightarrow ce^4=1$$

$$\Leftrightarrow c=e^{-4}$$

b. Find the marginal pdf's of X and Y.

$$f_X(x) = \sum_y f(x,y)$$

$$= \sum_y \left(e^{-4} \frac{2^{x+y}}{x!\,y!} \right)$$

$$= \sum_y \left(e^{-4} \frac{2^x}{x!} \frac{2^y}{y!} \right)$$

$$= e^{-4} \frac{2^x}{x!} \sum_y \frac{2^y}{y!}$$

$$= e^{-4} . \frac{2^x}{x!} . e^2$$

$$= e^{-2} \frac{2^x}{x!}, untuk\ x = 0,1,2,\ldots$$

$$
\begin{aligned}
f_Y(y) &= \sum_x f(x,y) \\
&= \sum_x \left(e^{-4}\, \frac{2^{x+y}}{x!\,y!} \right) \\
&= \sum_x \left(e^{-4}\, \frac{2^x}{x!}\, \frac{2^y}{y!} \right) \\
&= e^{-4}\, \frac{2^y}{y!} \sum_x \frac{2^x}{x!} \\
&= e^{-4} . \frac{2^y}{y!} . e^2 \\
&= e^{-2}\, \frac{2^y}{y!} \text{ ' } y = 0,1,2\ldots
\end{aligned}
$$

9.

		X_2		
		1	2	3
X_1	1	1/12	1/6	0
	2	0	1/9	1/5
	3	1/18	1/4	2/15

a. Find the marginal pdf's of X_1 dan X_2

i) $f_{X_1}(x_1) = \sum_{x_2} f(x_1, x_2)$

$$f_{X_1}(1) = \frac{1}{12} + \frac{1}{6} + 0 = \frac{3}{12} = \frac{1}{4}$$

$$f_{X_1}(2) = 0 + \frac{1}{9} + \frac{1}{5} = \frac{5+9}{45} = \frac{14}{45}$$

$$f_{X_1}(3) = \frac{1}{18} + \frac{1}{4} + \frac{2}{15}$$

ii) $f_{X_2}(x_1) = \sum_{x_1} f(x_1, x_2)$

$$f_{X_2}(1) = \frac{1}{12} + 0 + \frac{1}{18}$$

$$f_{X_2}(2) = \frac{1}{6} + \frac{1}{9} + \frac{1}{4}$$

$$f_{X_2}(3) = 0 + \frac{1}{5} + \frac{2}{15}$$

Silahkan dikerjakan

(b) Are X_1 and X_2 independent? Why or why not?

(c) Find $P[X_1 \leqslant 2]$.

(d) Find $P[X_1 \leqslant X_2]$.

(e) Tabulate the conditional pdf's, $f(x_2 \mid x_1)$ and $f(x_1 \mid x_2)$.

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